Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Course: MPZ 3132-Engineering Mathematics IB **Assignment No.02 Academic Year – 2011/2012**

Instructions

- Answer all questions
- Write your address back of your answer scripts
- Use both sides of paper when you are doing assignment**.**
- Please send the answer scripts of your assignment **on or before the due date** to the following address.

  Course Coordinator – MPZ 3132

  Dept. of Mathematics & Philosophy of Engineering

  Faculty of Engineering Technology

  The Open University of Sri Lanka.

  Nawala, Nugegoda.

You can collect model answers from virtual class ***(www.ou.ac.lk)***

User name - student0 **Password – MPZ3132**

1. Prove that $\frac{1}{D+\alpha}f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$

1.1. Using the above formula find general solution of the following differential equations

1.1.1. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2e^{3x}.$

1.1.2. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 3sin5x.$

1.2. Using the above formula and $D - Operaror$ methods find the general solutions of the following system of differential equations

$$\frac{dx}{dt} = x + 3y + e^t$$

$$\frac{dy}{dt} = x - y + e^{4t}$$

2. The differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 58cosx - 4sinx$ has a trial function of the form $y_T = Acosx + Bsinx.$

2.1. Find the values of A and B

2.2. Find the general solution of the above differential equation.

2.3. Using the solution of above differential equation and the substitution $z = xy$ prove that the general solution of the differential equation

$x\frac{d^2y}{dx^2} + 2(x+1)\frac{dy}{dx} + (5x+2)y = 58cosx - 4sinx$ is

$y = \frac{1}{x}e^{-x}(pcos2x + qsin2x) + \frac{13}{x}cos\left(x + tan^{-1}\frac{12}{5}\right)$ where p and q are arbitrary constants.

3. Consider the differential equation $\frac{d^2y}{dx^2} + 4y = 20e^{-4x}$

3.1. Using a suitable trial function find a particular integral for the above differential equation.

3.2. Find the general solution of the above differential equation.

3.3. Using the solution of above differential equation and the substitution $z = x^3y$ prove that the general solution of the differential equation

$x^3\frac{d^2y}{dx^2} + 6x^2\frac{dy}{dx} + 2x(2x^2+3)y = 20e^{-4x}$ is $y = \frac{1}{x^3}(Acos2x + Bsin2x) - \frac{1}{x^3}e^{-4x}$ where A and B are arbitrary constants.

4. Define the Laplace transformation.

4.1. Find the Laplace transformations of the following functions

4.1.1. $f(x) = \begin{cases} 0 & if \quad 0 < x < 2 \\ 3x^2 & if \quad 2 \le x \end{cases}$.

4.1.2. $sin^3x.$

4.2. If the Laplace transformation of $f(x)$ is $L(f(x)) = F(s)$ prove that $L(e^{\alpha x}f(x)) = F(s-\alpha).$

Find the inverse Laplace transformations of the following functions

4.2.1. $F(s) = \frac{2s+3}{s^2+4s+13}$

4.2.2. $F(s) = \frac{s^2+4}{(s+1)(s^2+2)}.$

5. Prove that $L(x^n e^{-\alpha x}) = \frac{n!}{(x+\alpha)^{n+1}}$

5.1. Using the Laplace transformations solve the following differential equation

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = x^4 e^{3x}$ where $y(0) = 2$ and $\left(\frac{dy}{dx}\right)_{x=0} = 6.$

5.2. Using the Laplace transformation solve the following system of differential equations with initial conditions $x(0) = -1$ and $y(0) = 0$.

$$\frac{dx}{dt} = 6x - 3y + 8e^t \qquad \frac{dy}{dt} = 2x + y + 4e^t$$

END